Rising AP Calculus & AP Statistics Students Summer Work

You are expected to do the first set, *Problems for June*, in June and submit them by email to me (rvilarrubi@barrie.org) by July 1st. I will answer any questions you email by June 25th.

You are expected to do the second set, *Problems for July*, in July and submit them by email to me by August 1st. I will answer any questions you email by July 28th.

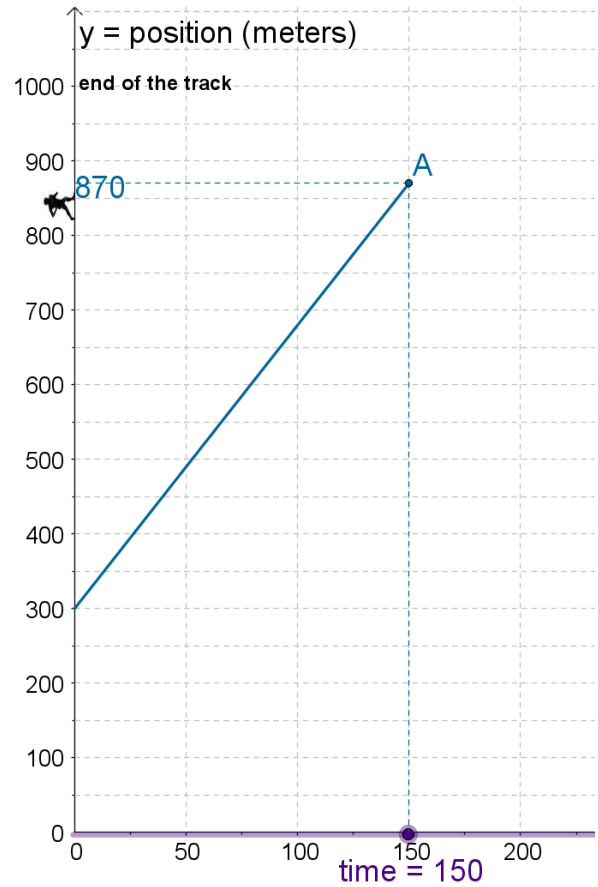
You are expected to do the third set, *Problems for August*, in August and submit them by email to me (rvilarrubi@barrie.org) by August 21th. I will answer any questions you email by August 19th.

Each of these sets, if responsibly done and submitted by the deadline will add 2 points to your first trimester grade

Problems for June

Problem 1 In the figure at right you can see the graph of the position function $P(t)$ of a runner. Position is measured in meters and time t in seconds

1a. What is the meaning of the coordinates (150, 870) of the point A in the context of the runner situation?



1b. What is the velocity of the runner?

1c. Find a rule for $P(t)$.

1d. What was the exact position of the runner at time $t = 80$?

1e. At what time will the runner reach the end of the track (position 1000)?

<u>Problem 2</u> ***Do not use technology for problems 2a or 2b.*** In each of the following cases, find the x- and y-intercepts (if any) and the vertical and horizontal asymptotes of the indicated quotient of linear functions. Then sketch its graph, labeling any important points. In each of the cases, used a sign chart to determine the regions to be shaded. Include the asymptotes using dashed lines. Work and graph tidily and clearly!!

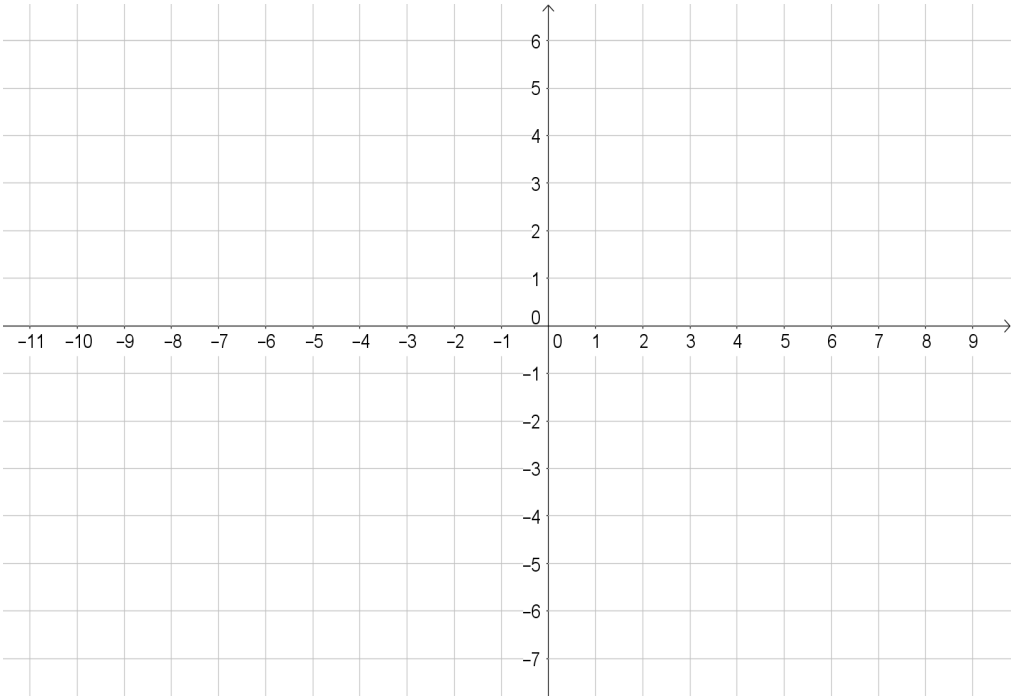
2a. $f(x) = \dfrac{2x-6}{x+2}$

Sign chart

Vertical Asymptote

Horizontal Asymptote

Find the y-intercept



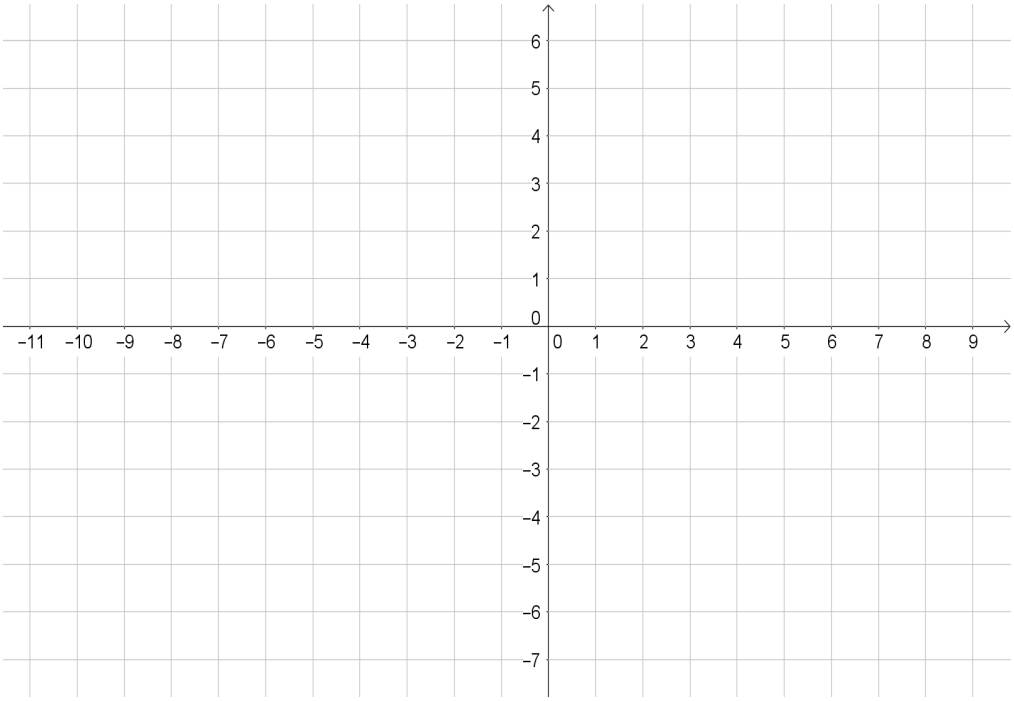
2b $f(x)=\dfrac{7+2x}{4-x}$

Sign chart

Vertical Asymptote

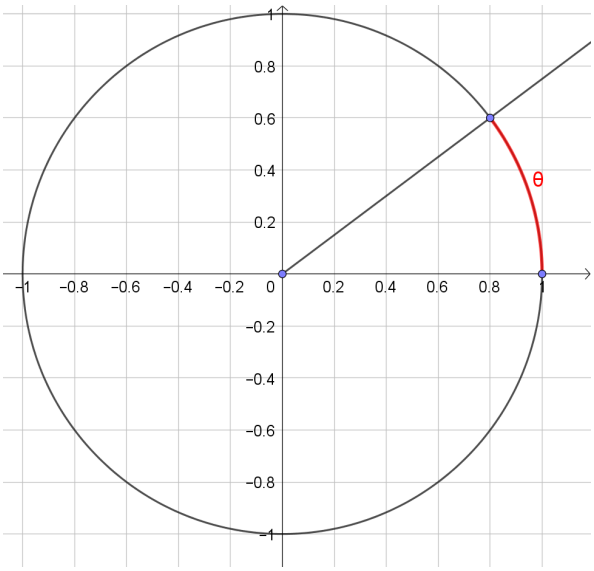
Horizontal Asymptote

Find the y-intercept



Problem 3

3a. Use the figure below to estimate $\sin\theta$ and $\cos\theta$



3b. Calculate each of the following

$\cos(\pi - \theta)$

$\sin(\pi + \theta)$

$\cos(-\theta)$

$\tan\theta$

<u>Problem 4</u> Simplify each of the following until you obtain a single exponential in base *b.* If this is impossible, explain what led you to that conclusion

4a. $\dfrac{b^{3x} \cdot b^{x-4}}{b^2} =$

4b. $\dfrac{\left(b^{x+1}\right)^2}{b} =$

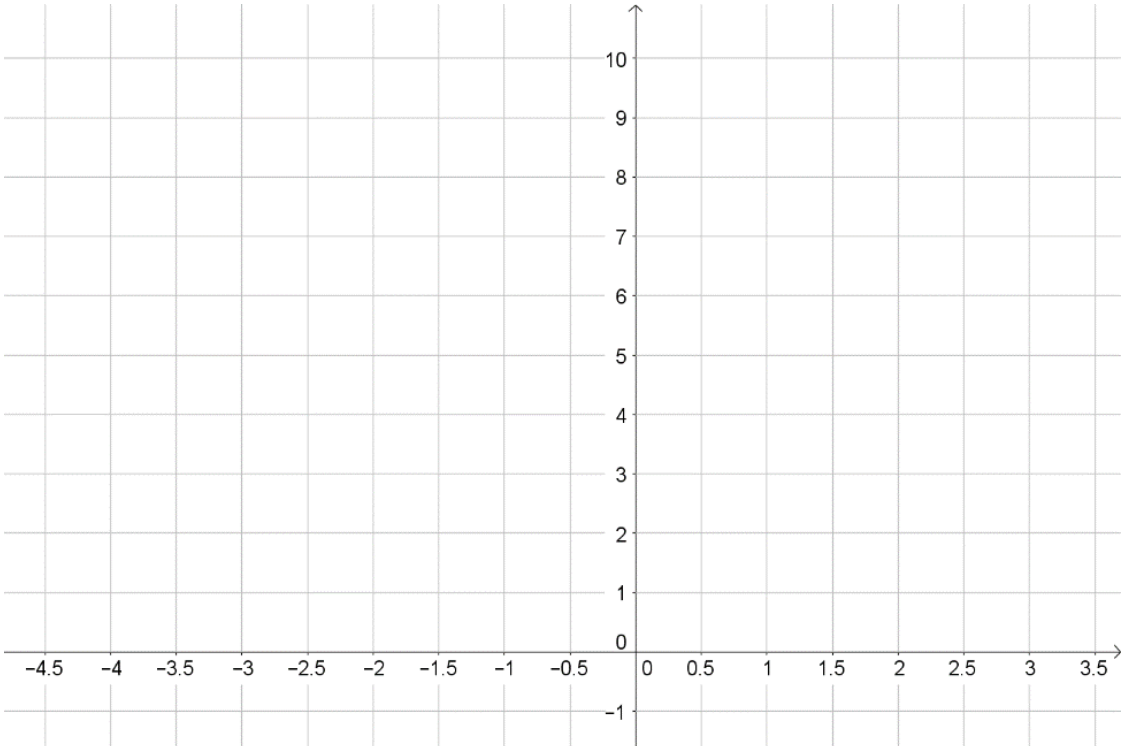
4c. $\dfrac{b^x + b^2}{b^3}$

4d. $\dfrac{b^2\sqrt{b}}{b^{-1}}$

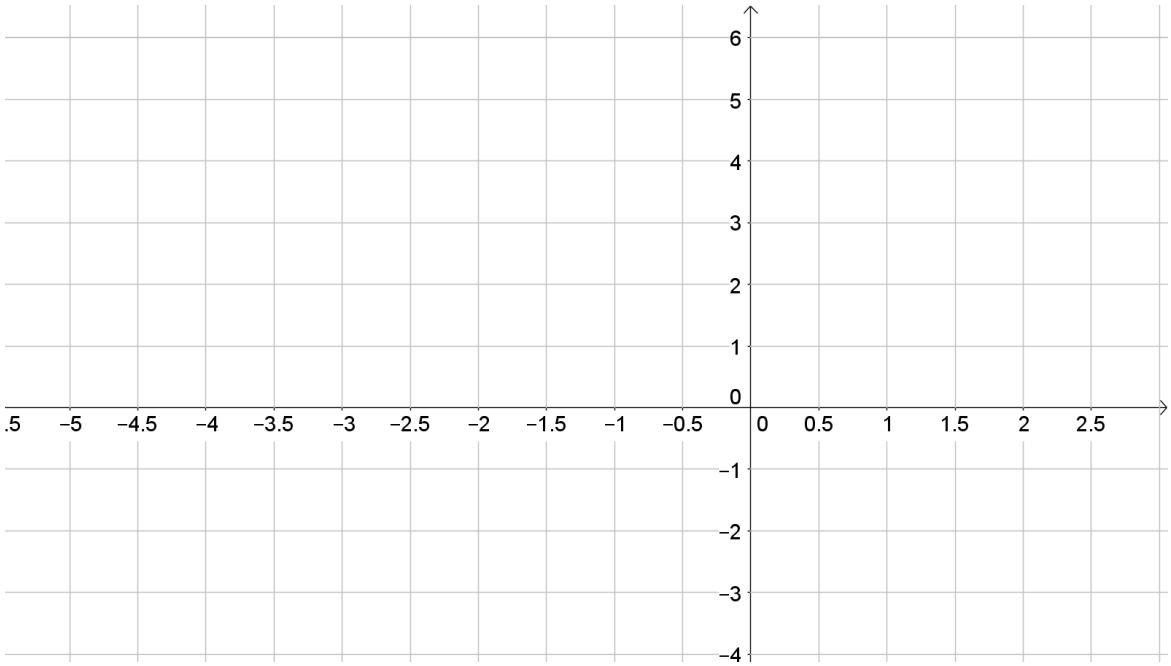
4e. $b\sqrt[3]{b\sqrt{b}}$

<u>Problem 5.</u> Graph each of the following functions ***without the aid of the graphing features of your calculator***. Explain the transformations and plot asymptotes clearly. In each case, include the values of the function for x (or t) = -1, 0, and 1, and make sure that there are no discrepancies.

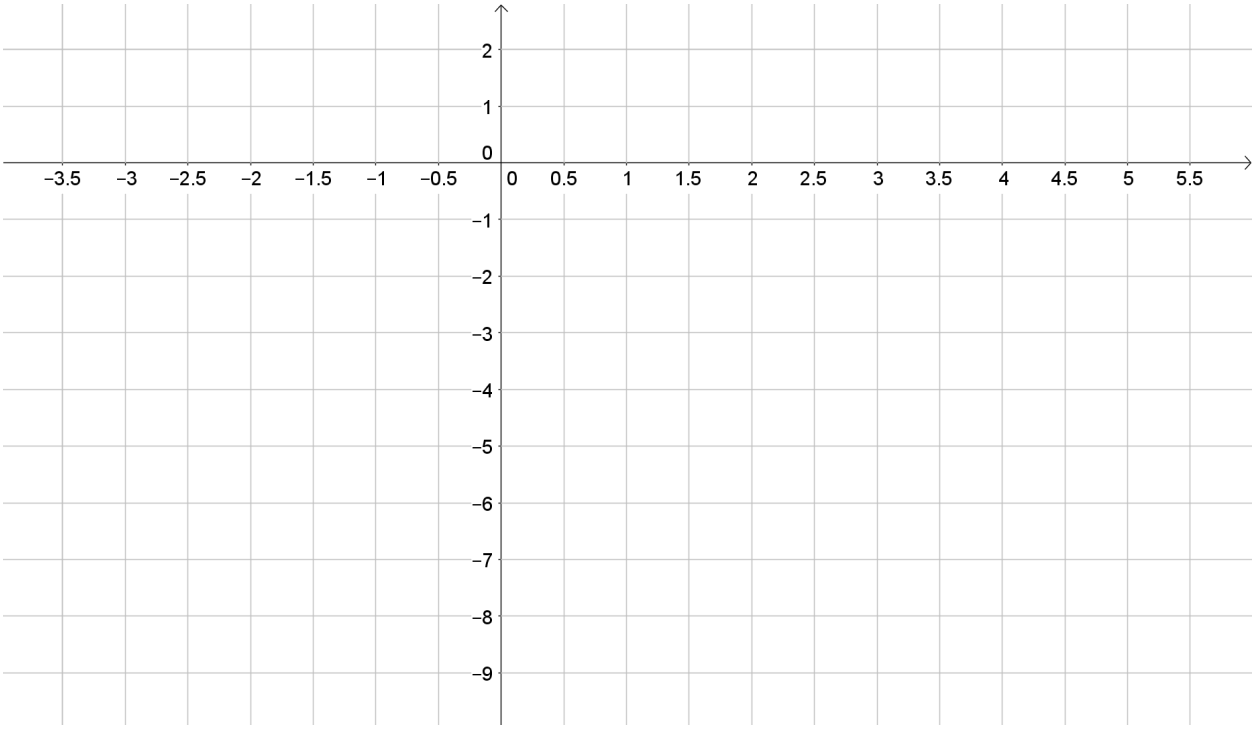
5a. $f(x) = 2 \cdot 2.5^x + 1$



5b. $g(t) = 1.5e^{t} - 3$ (find and label the x-intercept)



5c. $h(x) = -2 \cdot 0.4^x - 1$



Problem 6 Solve the following equations

6a. $(x^2 - 3x + 2)(x^2 - x + 5) = 0$

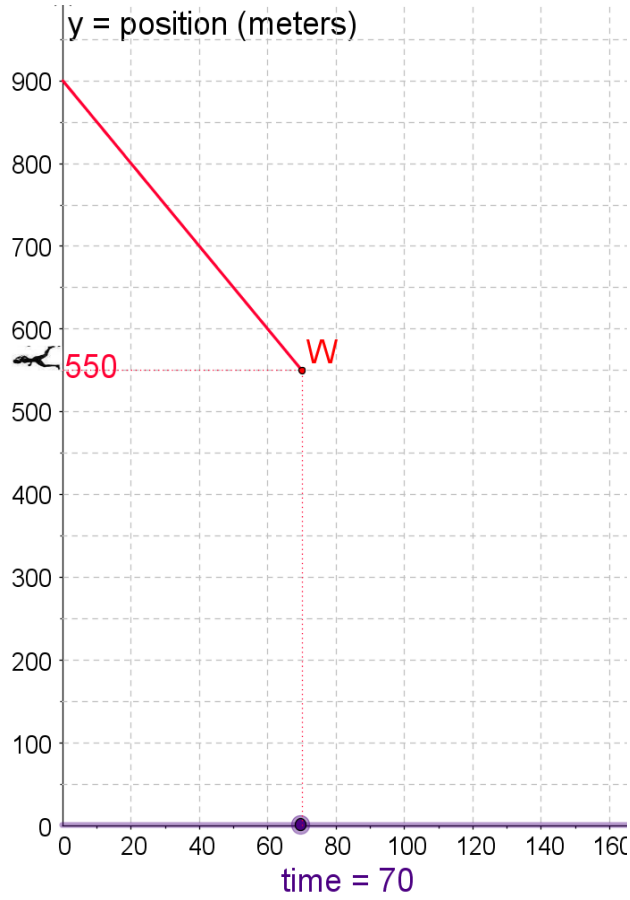
6b. $2x^3 - 8x^2 - 8x = 0$

6c. $\dfrac{3x - 7}{1 - x} = -2$

Problems for July

<u>Problem 1</u> In the figure at right you can see the graph of the position function $P(t)$ of a runner. Position is measured in meters and time t in seconds

What is the meaning of the coordinates (70, 550) of the point W in the context of the runner situation?



1b. What is the velocity of the runner?

1c. Find a rule for $P(t)$.

1d. What will the exact position of the runner be at time $t = 83$?

1e. At what time will the runner reach the end of the track (position 0)?

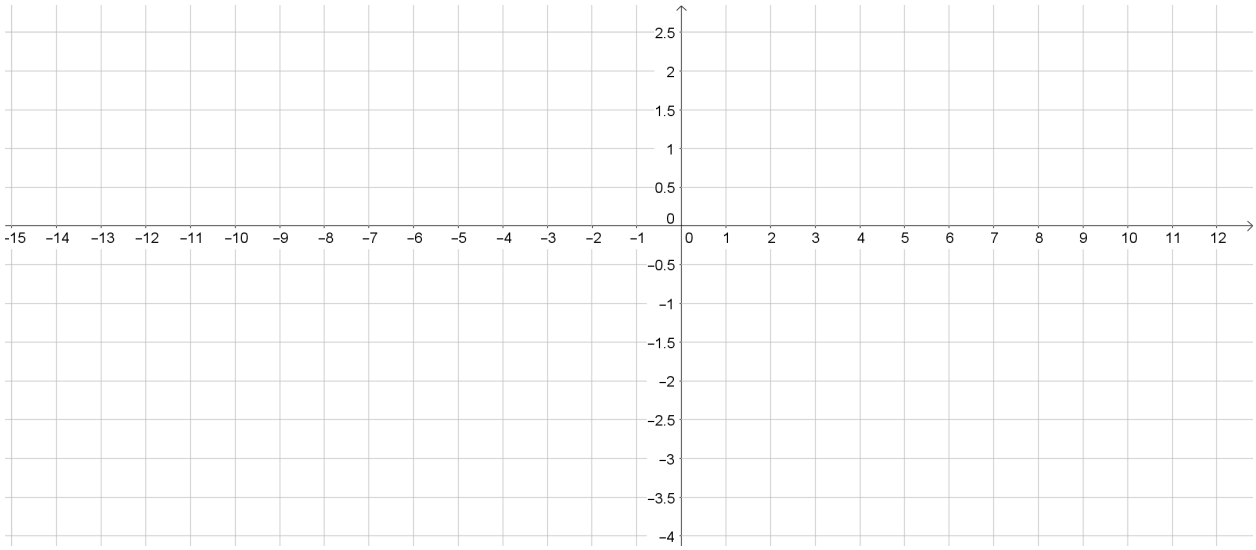
<u>Problem 2</u> ***Do not use technology for problems 2a or 2b.*** In each of the following cases, make a sign chart, shade the regions, plot x-intercepts and behavior surrounding the vertical asymptotes. Find and plot the horizontal asymptote. Then complete the sketch of the graph. Work and graph tidily and clearly.

2a. $$f(x) = \frac{x^2 - 3x - 10}{(2x-4)(x+6)}$$

Sign chart

Horizontal asymptote

y-intercept

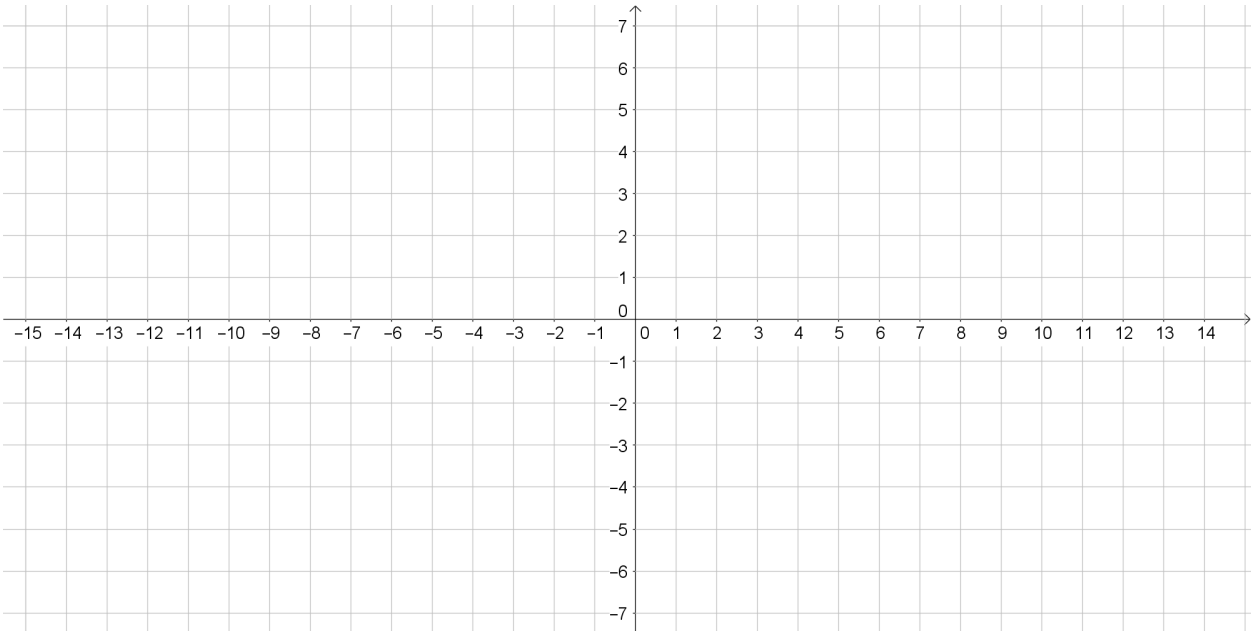


2b. $$f(x) = \frac{2(x^2 - 4x + 4)}{(-x^2 + 36)}$$

Sign chart

Horizontal asymptote

y-intercept



<u>Problem 3</u> (*You will need a graphing calculator for this problem*) The population of France is currently (2020) estimated to be 67.8 million and is increasing at an annual rate of 0.35%. We will assume that this rate of growth remains unchanged

3a. Write a recursive rule for the population of France as a function of time.

3b. Find an explicit rule for the population $P(t)$ of France t years from now.

3c. Use one of those rules to estimate the population of France in 2025.

3d. In approximately what year will the population of France reach 70 million?

3e. The United Kingdom population is currently about 65.8 million, slightly lower than the population of France, but is growing at rate of 0.5%. Assuming this rate of growth does not change, in what year, approximately, will the populations of these two countries be the same?

<u>Problem 4</u> Solve the following equations

4a. $2 \cdot e^{x} - 3 = 29$

4b. $4 \cdot e^{2x} + 8 = 56$

4c. $e^{x}(e^{x} - 4)(e^{x} + 7) = 0$

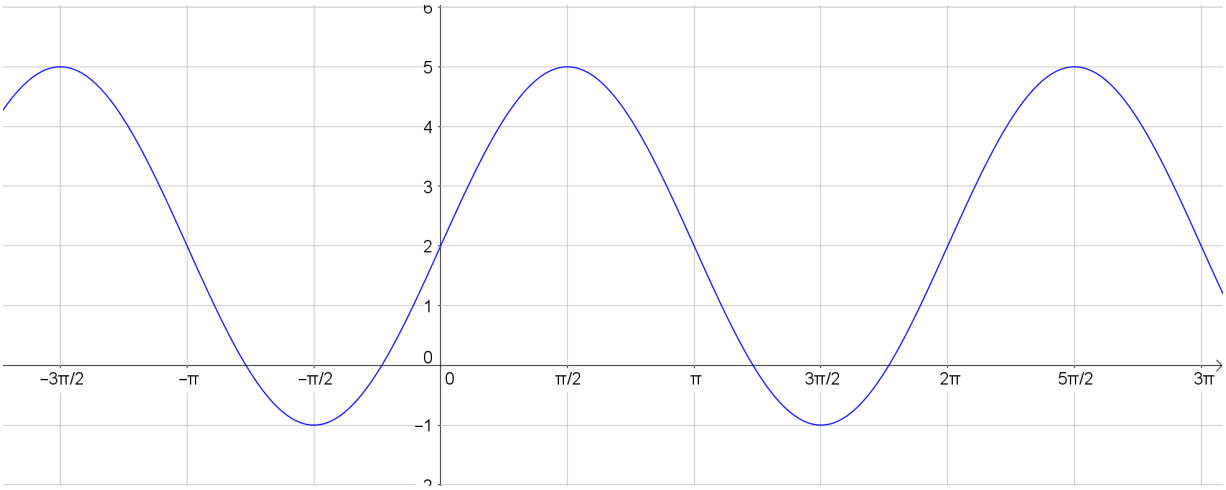
4d $3(\sin x - 2) = -4.2$

4e. $5\cos^2 x + 8\cos x - 4 = 0$

4f. $(4\sin^2 x - 3)(\cos x + 0.7) = 0$

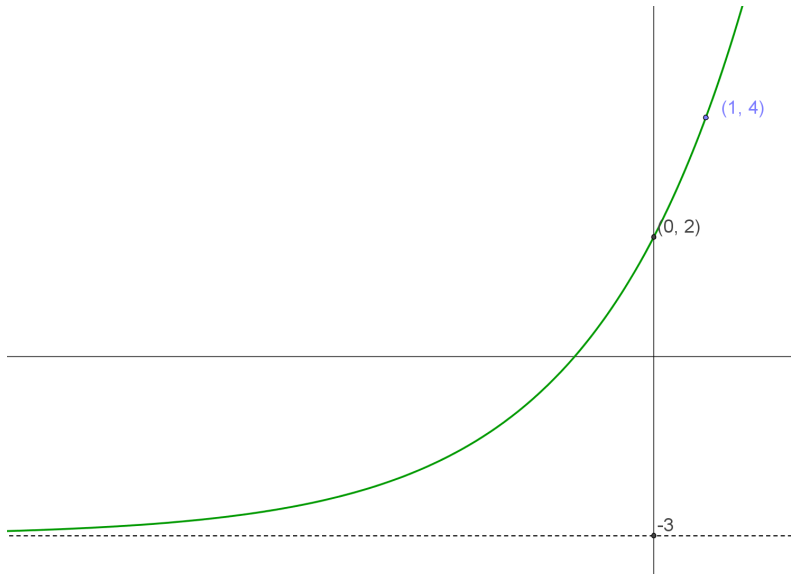
<u>Problem 5</u> Find a plausible rule for each of the functions graphed below

5a.

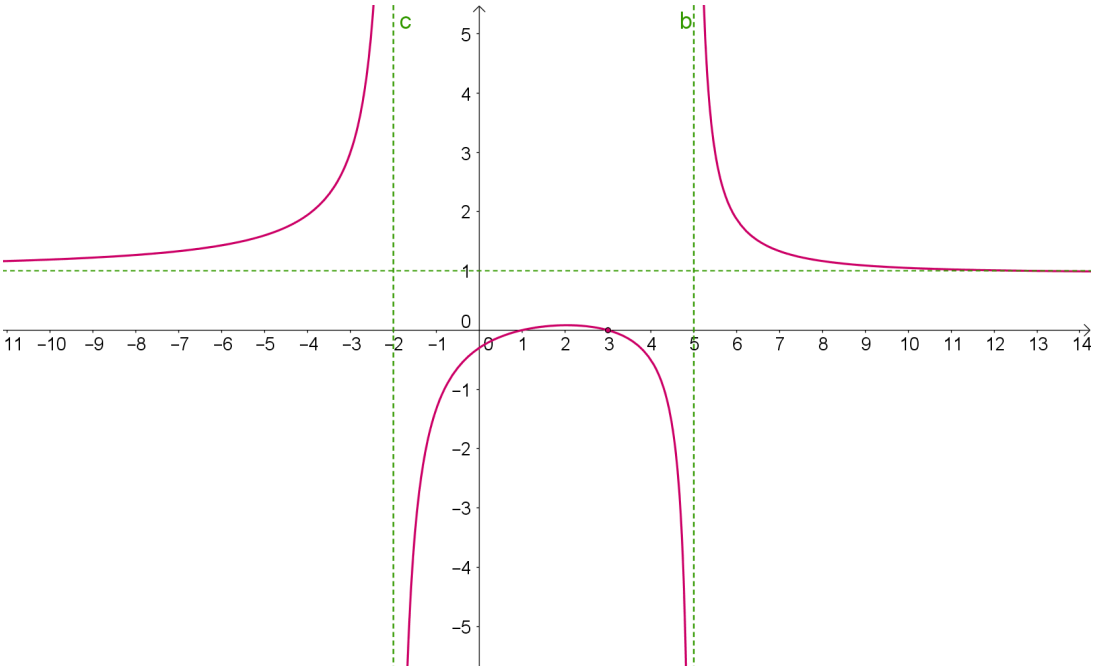


5b

5c.


(1, 4)
(0, 2)
-3

5d.


c
b
5
4
3
2
1
0
−1
−2
−3
−4
−5
11 −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Problems for August

<u>Problem 1</u> ***Do not use technology for problems 1a or 1b.*** In each of the following cases, make a sign chart, shade the regions, plot x-intercepts and behavior surrounding the vertical asymptotes. Calculate limits at the "common zeros" (factor/simplify/recalculate) and plot and label any bubbles. Find and plot the horizontal asymptote. Then complete the sketch of the graph. Work and graph tidily and clearly.

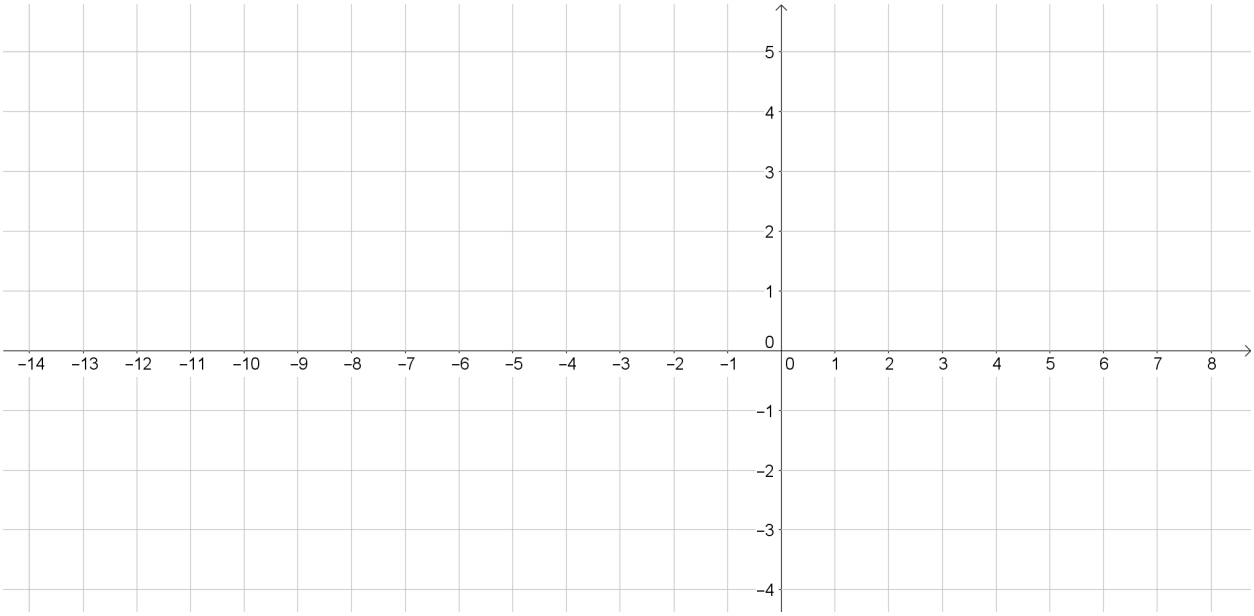
1a. $$f(x) = \frac{x^2 - 10x + 21}{(x+5)(x-3)}$$

Sign chart

Limit at common zeros

Horizontal asymptote

y-intercept



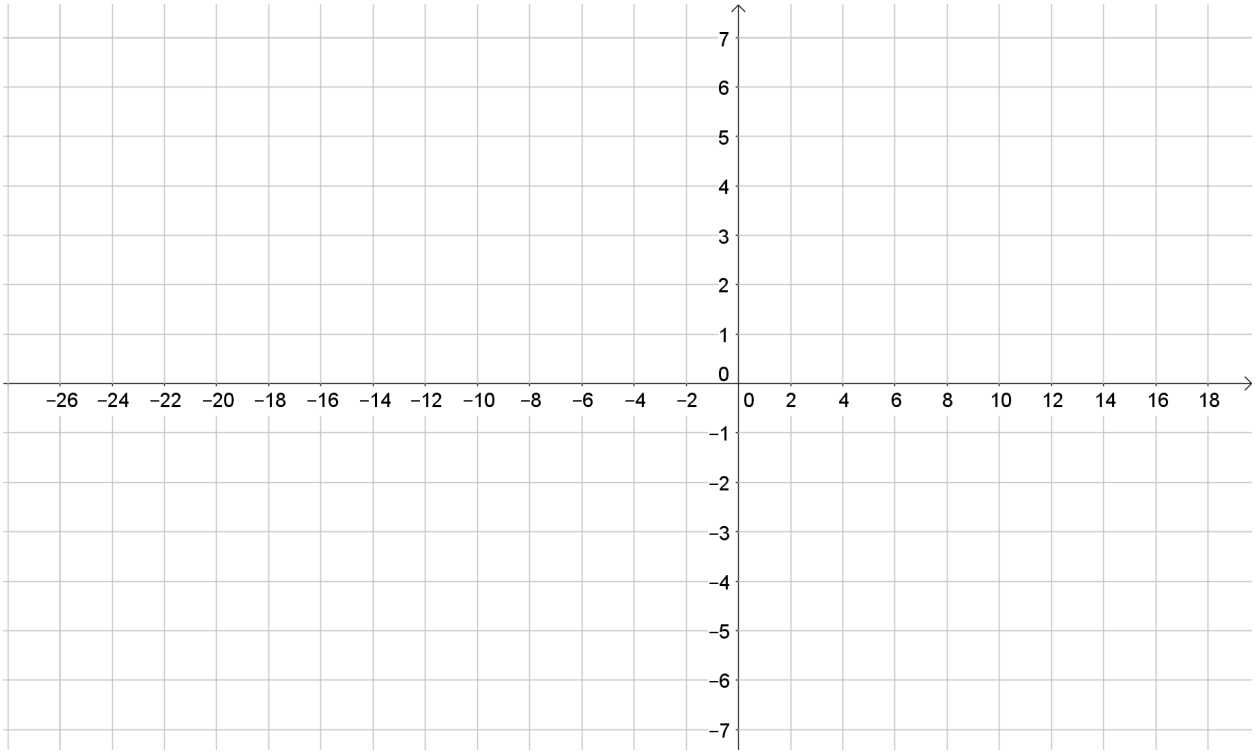
1b $f(x)=\dfrac{-2x^2+18x-36}{x^2-2x-24}$

Sign chart

Limit at common zeros

Horizontal asymptote

y-intercept


7
6
5
4
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−2
−3
−4
−5
−6
−7
−26 −24 −22 −20 −18 −16 −14 −12 −10 −8 −6 −4 −2 0 2 4 6 8 10 12 14 16 18

<u>Problem 2</u> Solve the equations below without using the graphing features of your calculator

2a. $-4(\tan x - 3)^2 + 6 = 22$

2b $\cos^2 x + 5\sin^2 x - 4\sin x = 0$

2c. $\ln|2x-4|=0$

2d. $-\frac{1}{2}\left(e^{x}-3\right)^{2}-1=-9$

<u>Problem 3</u> (*You will need a graphing calculator for this problem*) The population of Lithuania is currently estimated to be about 2.3 million and is decreasing at an annual rate of 1.1%. Assume that this rate of change will not change.

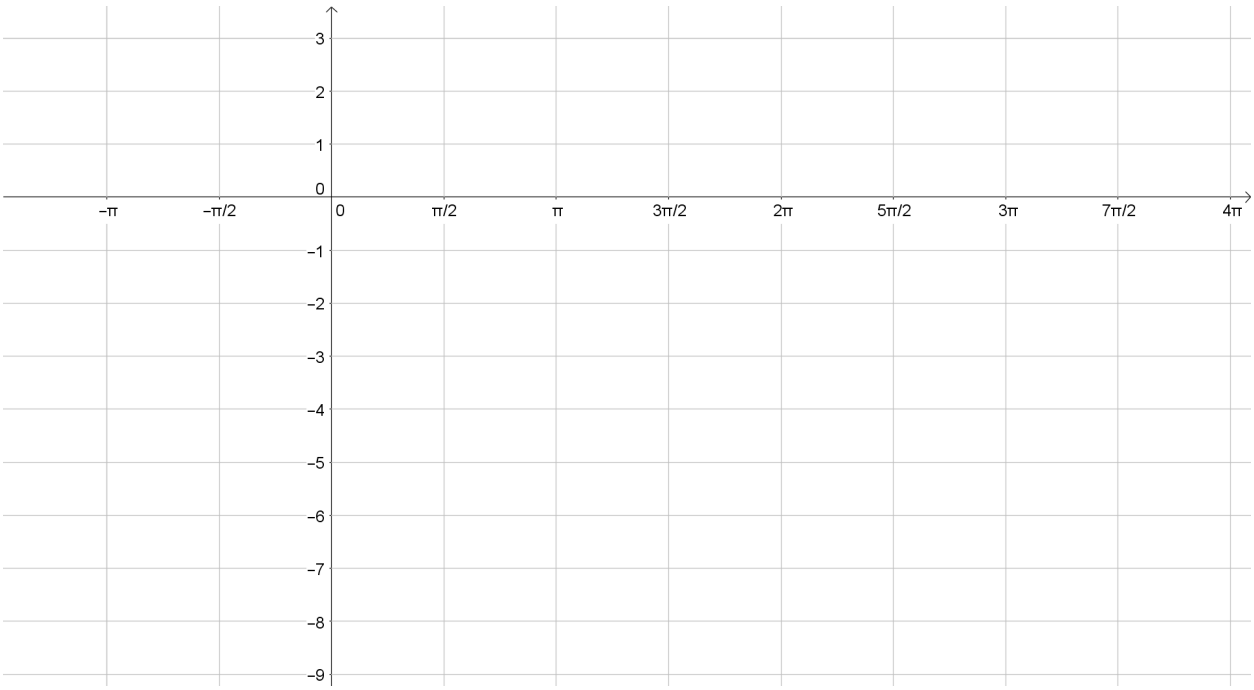
3a. Find a recursive rule for the population $P(t)$ of Lithuania t years from now

3b. Write an explicit rule for $P(t)$

3c. How long, approximately, will it take for the population of Lithuania to reach 2.5 million

Problem 4

4a. Graph the function $f(\theta) = 4\cos\left(\theta - \frac{\pi}{4}\right) - 3$ without the aid of technology. Remember to find and label minima (valleys), maxima (peaks), and crossings of the midline, as well as x and y intercepts. Show your work on this page.



4b. Graph the function $f(\theta) = -5\sin\left(\theta + \frac{\pi}{4}\right) + 2$ without the aid of technology. Remember to find and label minima (valleys), maxima (peaks), and crossings of the midline, as well as x and y intercepts. Show your work on this page.